

Effective Chiral Meson Lagrangian For The Extended Nambu – Jona-Lasinio Model

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Abstract

We present a derivation of the low-energy effective meson Lagrangian of the extended Nambu – Jona-Lasinio (ENJL) model. The case with linear realization of broken $SU(2) \times SU(2)$ chiral symmetry is considered. There are two crucial points why this revision is needed. Firstly it is the explicit chiral symmetry breaking effect. On the basis of symmetry arguments we show that relevant contributions related with the current quark mass terms are absent from the effective Lagrangians derived so far in the literature. Secondly we suggest a chiral covariant way to avoid non-diagonal terms responsible for the pseudoscalar – axial-vector mixing from the effective meson Lagrangian. In the framework of the linear approach this diagonalization has not been done correctly. We discuss as well the $SU(2) \times SU(2)/SU(2)$ coset space parametrization for the revised Lagrangian (nonlinear ansatz). Our Lagrangian differs in an essential way from those that have been derived till now on the basis of both linear and nonlinear realizations of chiral symmetry.

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1 Introduction

The Nambu – Jona-Lasinio (NJL) model [1] is useful because it allows to derive the effective meson Lagrangian from a more fundamental, i.e. microscopic, theory of quarks. The effective four-fermion interactions of the NJL-like models represent “certain approximations” to QCD. From the theoretical point of view, however, it is still not clear in which way these four-quark interactions arise in QCD. In the case of two flavours one of the possible mechanisms might be the quarks’ interaction via the zero modes of instantons [2], the so-called ’t Hooft interactions. Nevertheless there are a lot of investigations directed to the low-energy hadron phenomenology following from NJL-like Lagrangians [3]-[18]. The reasons are clear and well-known. These approximations are much easier to handle than QCD. They provide us with an unique way of constructing effective meson Lagrangians including vector and axial-vector mesons. They incorporate most of the short-distance relations which follow from QCD. In addition the NJL-like models are a good playground from the mathematical point of view. Starting from the basic quark Lagrangian one can develop both the techniques of the linear [3, 4, 7] and nonlinear [15] realizations of chiral symmetry. Both parametrizations for the chiral fields must lead to the same predictions and are equivalent on the mass-shell. The integration over the quark fields in the generating functional yields the determinant of the Dirac operator D in the presence of bosonic fields. Its evaluation must conform with the chiral covariant formulation of quantum field theory. The difficulties encountered in the realization of this idea are reviewed in [19].

In order to calculate the effective action and study spontaneous breakdown of global chiral symmetry it is important to employ a method of calculation which preserves the symmetry explicitly. It is known that the Schwinger proper-time representation [20, 21] for $\ln |\det D|$ in terms of the modulus of the quark determinant and the following long wavelength expansion of its heat kernel fulfills this requirement. This technique is especially good to describe the low-energy regime of QCD [22]. However, in the presence of the explicit chiral symmetry breaking term in the Lagrangian, the standard definition of $\ln |\det D|$ in terms of a proper-time integral

$$\ln |\det D| = -\frac{1}{2} \int_0^\infty \frac{dT}{T} \rho(T, \Lambda^2) \text{Tr} \left(e^{-TD^\dagger D} \right) \quad (1)$$

modifies the explicit chiral symmetry breaking pattern of the original quark Lagrangian and needs to be corrected in order to lead to the fermion determinant whose transformation properties exactly comply with the symmetry content of the basic Lagrangian [23]. The necessary modifications can be done by adding a functional in the collective fields and their derivatives to the definition of the real part of the fermion determinant, i.e. we define that

$$\text{Re}(\ln \det D) = \ln |\det D| + P. \quad (2)$$

In the limit $\hat{m} = 0$, where \hat{m} is a current quark mass, $P = 0$ and the old result (1) emerges as a part of our definition. This strategy reminds Gasser and Leutwyler's correcting procedure which they used however for a different purpose, namely to restore the standard result (1) for the real part of the fermion determinant defined by the heat kernel $\text{Tr}[\exp(-T\bar{D}^2)]$, especially chosen to include anomalies [22]. Both of these procedures are aimed at the subtraction of inessential contributions inherent to the starting definitions of $\det D$. These contributions are inessential in the sense that they change the content of the theory, what should not be. The procedures in [22] and ours differ however through the way of fixing the form of the functional P , because the origin of these contributions is different. In the case under consideration the functional P must be chosen in such a manner that the real part of the effective Lagrangian for the bosonized ENJL model \mathcal{L}_{eff} will have the same transformation laws as the basic quark Lagrangian \mathcal{L} . In addition it should not change the “gap”-equation, i.e. the Schwinger – Dyson equation which defines the vacuum state of the model. These requirements together completely fix the freedom inherent to the definition of this functional. Let us stress that in our case P cannot be fixed by the requirement that the determinant remains unchanged when axial-vector and pseudoscalar fields are switched off, like, for instance, in [22]. As a consequence P contributes to the effective potential of the NJL model at every step of the heat kernel expansion. We have P being a functional as opposed to a polynomial in [22]. This is a general feature related to the non-renormalizability of the NJL model. Using formula (2) together with the way we propose to fix P , one can systematically take into account the effect of explicit chiral symmetry breaking in the ENJL model. To show this is one of the reasons for this paper. The correct description of explicit chiral symmetry breaking is evidently necessary in order to obtain realistic mass formulae and meson dynamics. We derive these expressions here and show that they are different (already in the

leading current quark mass dependent part) from the results known in the literature.

The second reason for this work is related to the problem of the pseudoscalar – axial-vector mixing in the ENJL model. For some reason this diagonalization has never been done correctly in the framework of the linear realization of chiral symmetry, as it has been already indicated in [24]. The usual procedure recurs to a linearized transformation

$$a_\mu \rightarrow a_\mu + c\partial_\mu\pi \quad (3)$$

which ruins the chiral transformation properties of the field a_μ and gives rise to all sorts of apparent symmetry breaking. For example, it leads to the $\rho\pi\pi$ coupling of the form $\rho_\mu[\pi, \partial_\mu\pi]$, which breaks chiral symmetry. Here we suggest instead a covariant way to avoid non-diagonal terms responsible for the pseudoscalar – axial-vector mixing in the effective meson Lagrangian. The covariant redefinition of the axial-vector field cannot be done without a corresponding change in its chiral partner, i.e. the vector field. This is a direct consequence of the linear realization of chiral symmetry. We have found two bilinear combinations of scalar and pseudoscalar fields which transform like axial-vector and vector fields and are chiral partners at the same time. We also show that our procedure, if one rewrites it in the new coset space variables corresponding to the nonlinear representation of the chiral group, is identical to the one already known from [15] or [25].

As a result we get the effective meson Lagrangian of the ENJL model in a form which includes only the first three (a_0, a_1, a_2) Seeley – DeWitt coefficients in the asymptotic expansion for the heat kernel. We restrict to this approximation, the extension is straightforward. Because of the aforementioned reasons we obtain a new revised Lagrangian which obeys all symmetry requirements of the model for the case of linear realization of broken $SU(2) \times SU(2)$ chiral symmetry. We derive as well the $SU(2) \times SU(2)/SU(2)$ coset space parametrization for the revised Lagrangian. For that purpose the Lagrange multiplier method is used to eliminate the scalar field from the generating functional, thus arriving to the nonlinear version of the model.

The plan of the paper is the following: In Sec.2 we discuss the Lagrangian of ENJL model and show that chiral $SU(2) \times SU(2)$ transformations of quark fields dictate the transformation laws of the auxiliary bosonic fields. These collective variables are necessary to rearrange the four-quark Lagrangian of

the ENJL model in an equivalent Lagrangian which is only quadratic in the quark fields. In Sec.3 we show how to define the fermion determinant for the case in which explicit symmetry breaking takes place. We calculate the first three contributions in the asymptotic expansion of the heat kernel in full detail. We derive the corresponding correcting polynomial from the functional P and show that it is completely fixed by the symmetry breaking pattern of the basic quark Lagrangian and the requirement that P should not change the “gap”- equation. The effective meson Lagrangian \mathcal{L}_{eff} is obtained at the end of this section. In Sec.4 we introduce the new variables for vector and axial-vector fields in order to avoid the pseudoscalar – axial-vector mixing term from \mathcal{L}_{eff} . We use chiral covariant combinations for this replacement. We discuss the field renormalizations needed to define the physical meson states and the meson mass spectrum. The transition to the nonlinear version is done in Sec.5. The concluding remarks are given in Sec.6. Finally we show in the Appendix that the replacements of variables done in Sec.4 for the spin one mesons are completely equivalent to the replacement which has been already used in the literature in the context of the non-linear parametrization in the chiral group space.

2 Lagrangian and its symmetries

Consider the effective quark Lagrangian of strong interactions which is invariant under a global colour $SU(N_c)$ symmetry

$$\begin{aligned}\mathcal{L} = & \bar{q}(i\gamma^\mu\partial_\mu - \hat{m})q + \frac{G_S}{2}[(\bar{q}q)^2 + (\bar{q}i\gamma_5\tau_i q)^2] \\ & - \frac{G_V}{2}[(\bar{q}\gamma^\mu\tau_i q)^2 + (\bar{q}\gamma^\mu\gamma_5\tau_i q)^2].\end{aligned}\tag{4}$$

Here q is a flavour doublet of Dirac spinors for quark fields $\bar{q} = (\bar{u}, \bar{d})$. Summation over the colour indices is implicit. We use the standard notation for the isospin Pauli matrices τ_i . The current quark mass matrix $\hat{m} = \text{diag}(m_u, m_d)$ is chosen in such a way that $m_u = m_d$. Without this term the Lagrangian (4) would be invariant under global chiral $SU(2) \times SU(2)$ symmetry. The coupling constants G_S and G_V have dimensions (Length)² and can be fixed from the meson mass spectrum.

The transformation law for the quark fields is the following

$$\delta q = i(\alpha + \gamma_5 \beta)q, \quad \delta \bar{q} = -i\bar{q}(\alpha - \gamma_5 \beta) \quad (5)$$

where parameters of global infinitesimal chiral transformations are chosen as $\alpha = \alpha_i \tau_i$, $\beta = \beta_i \tau_i$. Therefore our basic Lagrangian \mathcal{L} transforms according to the law

$$\delta \mathcal{L} = -2i\hat{m}(\bar{q}\gamma_5 q). \quad (6)$$

It is clear that nothing must destroy this symmetry breaking requirement of the model (we are not considering anomalies here).

Following the standard procedure we introduce colour singlet collective bosonic fields in such a way that the action becomes bilinear in the quark fields and the quark integration becomes trivial

$$\begin{aligned} Z = & \int \mathcal{D}q \mathcal{D}\bar{q} \mathcal{D}s \mathcal{D}p_i \mathcal{D}V_\mu^i \mathcal{D}A_\mu^i \exp \left\{ i \int d^4x [\mathcal{L} \right. \\ & \left. - \frac{1}{2G_S}(s^2 + p_i^2) + \frac{1}{2G_V}(V_{\mu i}^2 + A_{\mu i}^2) \right] \Big\}. \end{aligned} \quad (7)$$

We suppress external sources in the generating functional Z and assume summation over repeated Lorentz (μ) and isospin ($i = 1, 2, 3$) indices. One has to require from the new collective variables that

$$\delta(s^2 + p_i^2) = 0, \quad \delta(V_{\mu i}^2 + A_{\mu i}^2) = 0 \quad (8)$$

in order not to destroy the symmetry of the basic Lagrangian \mathcal{L} . After replacement of variables in Z

$$s = \sigma - \hat{m} + G_S(\bar{q}q), \quad (9)$$

$$p_i = \pi_i - G_S(\bar{q}i\gamma_5\tau_i q), \quad (10)$$

$$V_\mu^i = v_\mu^i + G_V(\bar{q}\gamma_\mu\tau_i q), \quad (11)$$

$$A_\mu^i = a_\mu^i + G_V(\bar{q}\gamma_\mu\gamma_5\tau_i q), \quad (12)$$

these requirements together with (5) lead to the transformation laws for the new collective fields

$$\delta\sigma = -\{\beta, \pi\}, \quad \delta\pi = i[\alpha, \pi] + 2(\sigma - \hat{m})\beta, \quad (13)$$

$$\delta v_\mu = i[\alpha, v_\mu] + i[\beta, a_\mu], \quad \delta a_\mu = i[\alpha, a_\mu] + i[\beta, v_\mu]. \quad (14)$$

We have introduced the notation $\pi = \pi_i \tau_i$, $v_\mu = v_{\mu i} \tau_i$, $a_\mu = a_{\mu i} \tau_i$. Therefore the transformation law of the quark fields finally defines the transformation law of the bosonic fields.

The Lagrangian in the new variables ($\mathcal{L} \rightarrow \mathcal{L}'$) has the form

$$\mathcal{L}' = \bar{q} D q - \frac{(\sigma - \hat{m})^2 + \pi_i^2}{2G_S} + \frac{v_{\mu i}^2 + a_{\mu i}^2}{2G_V}. \quad (15)$$

where

$$D = i\gamma^\mu \partial_\mu - \sigma + i\gamma_5 \pi + \gamma^\mu (v_\mu + \gamma_5 a_\mu). \quad (16)$$

Let us note that although the Dirac operator D does not include the current quark mass, \hat{m} , the transformation law of pion fields does. Thus,

$$\delta D = i[\alpha, D] - i\{\gamma_5 \beta, D\} - 2i\hat{m}\gamma_5 \beta; \quad (17)$$

i.e., the transformation law of the Dirac operator has an inhomogeneous term which is proportional to \hat{m} . In particular, we have

$$\delta(D^\dagger D) = i[\alpha + \gamma_5 \beta, D^\dagger D] + 2i\hat{m}(\gamma_5 \beta D - D^\dagger \gamma_5 \beta). \quad (18)$$

The second term $\sim \hat{m}$ can be used to get systematically the explicit symmetry breaking pattern of the effective Lagrangian derived on the basis of formula (1). The simplest way to do this is to work in Euclidean space. Since the Dirac γ -matrices in this space are antihermitian the combination proportional to the derivatives contained in $(\gamma_5 \beta D - D^\dagger \gamma_5 \beta)$ will vanish. It simplifies substantially the evaluation of $\delta \ln |\det D|$ and allows to derive in a closed form the functional P in (2). After that the asymptotic expansion of P to obtain the correcting polynomials at each power of the proper-time will be a purely technical procedure. However in this paper we prefer to work directly in Minkowski space and present an alternative way to derive correcting polynomials step by step starting from the first term of the proper-time expansion.

The subsequent integration over quark fields shows that the effective potential has a non-trivial minimum and that spontaneous chiral symmetry breaking takes place. Redefining the scalar field $\sigma \rightarrow \sigma + m$ we come finally to the effective action

$$S_{\text{eff}} = -i \ln \det D_m - \int d^4 x \left[\frac{(\sigma + m - \hat{m})^2 + \pi_i^2}{2G_S} - \frac{v_{\mu i}^2 + a_{\mu i}^2}{2G_V} \right] \quad (19)$$

where the Dirac operator D_m is equal to

$$D_m = i\gamma^\mu \partial_\mu - m - \sigma + i\gamma_5 \pi + \gamma^\mu (v_\mu + \gamma_5 a_\mu). \quad (20)$$

In this broken phase the transformation law of the pion field changes to

$$\delta\pi = i[\alpha, \pi] + 2(\sigma + m - \hat{m})\beta \quad (21)$$

in full agreement with the variable replacement $\sigma \rightarrow \sigma + m$ for the scalar field in (13). What remains to be done to have an explicit representation of the effective action to leading order in the low energy expansion is to evaluate the determinant of the differential operator D_m . We shall consider this problem in the following section.

3 Calculation of the real part of the fermion determinant: the current quark mass effect

The modulus of the fermion determinant, $\ln |\det D_m|$, is conveniently calculated using the heat kernel method or even more directly in the way suggested in [19]. The result of these calculations on the basis of formula (1) is well known, see for example [7]. We prefer this way to the direct calculation of Feynman one-loop integrals [26, 27, 4, 10], since we need a method which allows to control the symmetry content of the result at each considered step. The differential operator D_m depends on collective meson fields which have well defined transformation laws with respect to the action of the chiral group. If one neglects the current quark mass term in the basic quark Lagrangian the combination $D_m^\dagger D_m$ transforms covariantly, i.e.

$$\delta(D_m^\dagger D_m) = i[\alpha + \gamma_5 \beta, D_m^\dagger D_m]. \quad (22)$$

This fact ensures that the definition of the real part of $\ln \det D_m$ in terms of the proper-time integral (1) cannot destroy the symmetry properties of the basic Lagrangian. However, if $\hat{m} \neq 0$ this is no longer true. There is no doubt that the current quark mass does break chirality in the definition $\ln |\det D_m|$, for we have seen in Sec.2 that the combination $D_m^\dagger D_m$ transforms inhomogeneously. The question is however, whether one should trust the result obtained through the formula $\ln |\det D_m|$. We have found that this definition needs to be corrected in the presence of the explicit symmetry breaking

term, since otherwise the transformation law of the effective bosonized meson Lagrangian will be different from the transformation law of the basic quark Lagrangian, i.e. the content of the theory will be changed. As we already mentioned in the introduction the problem can be solved if we define the real part of the fermion determinant through formula (2). This definition can be extended to include the case with the heat kernel suggested by Gasser and Leutwyler [22] or vice versa, for the formal part of these definitions is the same. The question how to extend the Gasser and Leutwyler's treatment of the chiral fermion determinant to the case of non-renormalizable models like NJL has been considered in [28]. Therefore we proceed from the definition

$$-i \ln \det D_m = \frac{i}{2} \int_0^\infty \frac{dT}{T} \rho(T, \Lambda^2) \text{Tr} \left(e^{-T \bar{D}_m^2} \right) - \int d^4x P(\sigma, \pi, v_\mu, a_\mu) \quad (23)$$

where P picks up all inessential contributions contained in the proper time integral including the terms with the explicit symmetry breaking. The operator \bar{D}_m is of the form $\bar{D}_m = \gamma_5 D_m$. At this level one should not worry that the expression \bar{D}_m^2 does not transform covariantly under the action of the chiral group. There is nothing wrong with this, as long as one is careful to express the final result in terms of chiral invariant quantities. The present procedure allows to do it in a systematic and consistent way for each order of the heat kernel expansion. The functional $P(\sigma, \pi, v_\mu, a_\mu)$ depends on collective fields and their derivatives. We define it by requiring the real part of the fermion determinant to transform as Lagrangian (4). The imaginary part of $\ln \det D_m$ will be discussed elsewhere. The expression (23) belongs to the ones which are known as proper-time regularizations. In the case of non-renormalizable models like ENJL we have to introduce the cutoff Λ to render the integrals over T convergent. We consider a class of regularization schemes which can be incorporated in the expression (23) through the kernel $\rho(T, \Lambda^2)$. These regularizations allow to shift in loop momenta. A typical example is the covariant Pauli-Villars cutoff [29]

$$\rho(T, \Lambda^2) = 1 - (1 + T\Lambda^2)e^{-T\Lambda^2}. \quad (24)$$

Let us put this expression into formula (1) and calculate the corresponding effective potential $V(\sigma, \pi_i)$, using eq.(16) with fields v_μ and a_μ switched off. We have as a result that

$$V(\sigma, \pi_i) = \frac{\hat{m}\sigma}{G_S} + \frac{\sigma^2 + \pi_i^2}{2G_S} \left(1 - \frac{N_c G_S \Lambda^2}{4\pi^2} \right)$$

$$+ \frac{N_c}{8\pi^2} \left[(\sigma^2 + \pi_i^2)^2 \ln \left(1 + \frac{\Lambda^2}{\sigma^2 + \pi_i^2} \right) - \Lambda^4 \ln \left(1 + \frac{\sigma^2 + \pi_i^2}{\Lambda^2} \right) \right]. \quad (25)$$

The minimum of this potential is localized at the point $\sigma = \langle \sigma \rangle_0 = m$ which is the solution of the “gap”-equation

$$\frac{m - \hat{m}}{mG_S} = \frac{N_c J_0}{2\pi^2}. \quad (26)$$

The function J_0 is one of the set of integrals J_n appearing in the result of the asymptotic expansion of (23)

$$J_n = \int_0^\infty \frac{dT}{T^{2-n}} e^{-Tm^2} \rho(T, \Lambda^2), \quad n = 0, 1, 2, \dots \quad (27)$$

Although the potential $V(\sigma, \pi_i)$ leads to the correct form of the “gap”-equation² [1], it is incomplete in its \hat{m} -dependent part. The reason is obvious: it destroys the symmetry breaking pattern of the basic Lagrangian, as one can conclude after short calculations. We did not include in (25) the corresponding part from the functional P .

Let us show how to get these counterterms on the basis of formula (23). We have for \bar{D}_m^2 the following representation

$$\bar{D}_m^2 = d^\mu d_\mu + m^2 + Q \quad (28)$$

where

$$\begin{aligned} d_\mu &= \partial_\mu + A_\mu, \quad A_\mu = \gamma^\mu \gamma_5 \pi - i v_\mu + \frac{i}{2} [\gamma^\nu, \gamma^\mu] \gamma_5 a_\nu, \\ Q &= \sigma^2 + 2m\sigma + 3\pi^2 - 2a^\mu a_\mu + i\gamma^\mu (\partial_\mu \sigma + 2\{a_\mu, \pi\}) \\ &\quad - \frac{1}{2} [\gamma^\mu, \gamma^\nu] (a_\mu a_\nu + v_\mu v_\nu + i\partial_\mu v_\nu) \\ &\quad - i\gamma_5 (\partial_\mu a^\mu + i[a_\mu, v^\mu] + 2(\sigma + m)\pi). \end{aligned} \quad (29)$$

The functional trace in formulae (23) is equal to

$$\text{Tr} \left(e^{-T\bar{D}_m^2} \right) = i \int d^4x \frac{e^{-Tm^2}}{(4\pi T)^2} \sum_{n=0}^{\infty} \text{tr}(T^n a_n) \quad (30)$$

²It is the same solution as the one from the Schwinger-Dyson equation.

where tr denote the traces over colour, flavour and Lorentz indices. The coefficients $a_n \equiv a_n(x, x)$ are the coincidence limit of Seeley – DeWitt coefficients. We need the first three of them for our purposes

$$a_0 = 1, \quad a_1 = -Q, \quad a_2 = \frac{1}{2}Q^2 + \frac{1}{12}F^2 \quad (31)$$

where $F^2 = F^{\mu\nu}F_{\mu\nu}$ and $F_{\mu\nu} = [d_\mu, d_\nu]$.

In [28] we have shown for $\hat{m} = 0$ how to obtain the Gasser and Leutwyler's part of functional P by which this definition of heat kernel needs to be modified in order to arrive at the fermion determinant whose real part is invariant under chiral transformations. Let us now show how to get the explicit symmetry breaking part P' of the functional P . Restricting to the second order Seeley – DeWitt coefficient one can obtain from (19) and (23) the effective Lagrangian

$$\begin{aligned} \mathcal{L}_{\text{eff}} = & \frac{v_{\mu i}^2 + a_{\mu i}^2}{2G_V} - \frac{1}{2G_S}[(\sigma + m - \hat{m})^2 + \pi_i^2] + \frac{N_c J_0}{4\pi^2}(\sigma^2 + 2m\sigma + \pi_i^2) \\ & - \frac{N_c J_1}{8\pi^2} \left[\frac{1}{6} \text{tr}(v_{\mu\nu}^2 + a_{\mu\nu}^2) - \frac{1}{2} \text{tr}((\nabla'_\mu \pi)^2 + (\nabla_\mu \sigma)^2) \right. \\ & \left. + (\sigma^2 + 2m\sigma + \pi_i^2)^2 \right] - P'(\sigma, \pi, v_\mu, a_\mu) \end{aligned} \quad (32)$$

where trace is to be taken in isospin space. In the considered approximation the functional P' is simply a polynomial. Here we have used the notation

$$v_{\mu\nu} = \partial_\mu v_\nu - \partial_\nu v_\mu - i[v_\mu, v_\nu] - i[a_\mu, a_\nu], \quad (33)$$

$$a_{\mu\nu} = \partial_\mu a_\nu - \partial_\nu a_\mu - i[a_\mu, v_\nu] - i[v_\mu, a_\nu], \quad (34)$$

$$\nabla_\mu \sigma = \partial_\mu \sigma - i[v_\mu, \sigma] + \{a_\mu, \pi\}, \quad (35)$$

$$\nabla'_\mu \pi = \partial_\mu \pi - i[v_\mu, \pi] - \{a_\mu, \sigma + m\}. \quad (36)$$

In formula (32) we already fixed the part of the functional P which is responsible for the chiral symmetric contribution. Now we need only to determine the explicit symmetry breaking part $P'(\sigma, \pi, v_\mu, a_\mu)$. If one uses the classical equation of motion for the pion field, $\pi_i = iG_S \bar{q}\gamma_5 \tau_i q$, see (15), one can rewrite eq.(6) in terms of meson fields

$$\delta\mathcal{L} = -\frac{2\hat{m}}{G_S}(\beta_i \pi_i). \quad (37)$$

Now our task is to choose the polynomial P' in such a way that the Lagrangian (32) will have the same transformation law. Let us note that P' is unique up to a chirally invariant polynomial. One can always choose P' in such a manner that the “gap”-equation is not modified, i.e. using this chiral symmetry freedom to avoid from P' terms linear in σ . One can do this noting that $\delta(\sigma^2 + \vec{\pi}^2) = -2(m - \hat{m})\delta\sigma$. It completely fixes the chiral freedom in P' . The variation of $P'(\sigma, \pi, v_\mu, a_\mu)$ has to cancel all terms which break explicitly chiral symmetry, excluding (37). As a result we get that

$$P'(\sigma, \pi, v_\mu, a_\mu) = \frac{\hat{m}^2(\sigma^2 + \pi_i^2)}{2m(m - \hat{m})G_S} - \hat{m}\frac{N_c J_1}{2\pi^2}[(2m - \hat{m})\sigma^2 + \sigma(\sigma^2 + \pi_i^2)] \\ + \hat{m}\frac{N_c J_1}{4\pi^2}\text{tr}\{(2m - \hat{m})a_\mu^2 - a_\mu\partial_\mu\pi + ia_\mu[v_\mu, \pi] + 2\sigma a_\mu^2\}. \quad (38)$$

Finally we have the following expression for the effective Lagrangian

$$\mathcal{L}_{\text{eff}} = \frac{v_{\mu i}^2 + a_{\mu i}^2}{2G_V} - \frac{\hat{m}(\sigma^2 + \vec{\pi}^2)}{2(m - \hat{m})G_S} - \frac{N_c J_1}{8\pi^2} \left[\frac{1}{6}\text{tr}(v_{\mu\nu}^2 + a_{\mu\nu}^2) \right. \\ \left. - \frac{1}{2}\text{tr}\left((\nabla_\mu\pi)^2 + (\nabla_\mu\sigma)^2\right) + \left(\sigma^2 + 2(m - \hat{m})\sigma + \pi_i^2\right)^2 \right] \quad (39)$$

where

$$\nabla_\mu\pi = \partial_\mu\pi - i[v_\mu, \pi] - \{a_\mu, \sigma + m - \hat{m}\}. \quad (40)$$

It can be verified, by explicit calculation, that the second term in this expression gives the correct behaviour of the Lagrangian with respect to chiral transformations. Other terms are combined in chiral invariant groups. For example, the terms proportional to J_1 are chiral invariant. The same will be true for each group of terms at the same J_n where $n \geq 1$. In particular one can in this way obtain systematically and step by step in the expansion (30) the \hat{m} -part of the effective potential which escaped from (25).

4 πa_μ mixing, field renormalizations and meson mass spectrum

Having established the effective Lagrangian one may look for kinetic and mass terms of the composite meson fields and extract the physical meson

masses by bringing the kinetic terms to the canonical form by means of field renormalizations. It is however known that for the axial-vector field a_μ one encounters the complication that chiral symmetry allows for terms which induce mixing between a_μ and pseudoscalar mesons. Such couplings must and can always be transformed away by a transformation which removes the spin-0 component of a_μ . The simplest replacement of variables which really fulfills the necessary transformation property (14) is

$$\begin{aligned} a_\mu &= a'_\mu + \frac{\kappa}{2} (\{\sigma + m - \hat{m}, \partial_\mu \pi\} - \{\pi, \partial_\mu \sigma\}), \\ v_\mu &= v'_\mu + \frac{i\kappa}{2} ([\sigma, \partial_\mu \sigma] + [\pi, \partial_\mu \pi]), \end{aligned} \quad (41)$$

reminiscent of the axial-vector and vector currents of the linear sigma model, [30]. One can see that these formulae include the linear part of formula (3) and do not lead to unwanted linear contributions in the vector field transformations (there is no any $\sigma - v_\mu$ mixing in this model). One can conclude that eq.(3) is just a piece of a more complicated expression which has to be used for a correct removing of the πa_μ mixing effect in this approach. In the case under consideration the commutator $[\sigma, \partial_\mu \sigma] = 0$. These new redefinitions, as compared to (3), will induce changes at the level of couplings with three or more fields. In the Appendix it is shown that the replacement (41) is identical to the field redefinition considered in [15] for the case of nonlinear realization of chiral symmetry. The constant κ is fixed by the requirement that the bilinear part of the effective Lagrangian becomes diagonal in the fields π, a'_μ . We find in this way that

$$\frac{1}{2\kappa} = (m - \hat{m})^2 + \frac{\pi^2}{N_c J_1 G_V}. \quad (42)$$

To define the physical meson fields let us consider the bilinear part of the effective Lagrangian

$$\begin{aligned} \mathcal{L}_{\text{free}} &= \frac{1}{4} \text{tr} \left\{ G_V^{-1} \left[(v'_\mu)^2 + (a'_\mu)^2 + \kappa^2 (m - \hat{m})^2 (\partial_\mu \pi)^2 - \frac{\hat{m}(\sigma^2 + \pi^2)}{m - \hat{m}} \right] \right. \\ &+ \frac{N_c J_1}{4\pi^2} \left[(\partial_\mu \sigma)^2 + g_A^2 (\partial_\mu \pi)^2 + 4(m - \hat{m})^2 [(a'_\mu)^2 - \sigma^2] \right. \\ &\left. \left. - \frac{1}{3} (\partial_\mu v'_\nu - \partial_\nu v'_\mu)^2 - \frac{1}{3} (\partial_\mu a'_\nu - \partial_\nu a'_\mu)^2 \right] \right\}. \end{aligned} \quad (43)$$

The following renormalizations lead to the standard form for the kinetic terms of spin-1 fields

$$v'_\mu = \sqrt{\frac{6\pi^2}{N_c J_1}} v_\mu^{(ph)} \equiv \frac{g_\rho}{2} v_\mu^{(ph)}, \quad a'_\mu = \frac{g_\rho}{2} a_\mu^{(ph)}. \quad (44)$$

Then we have

$$m_\rho^2 = \frac{6\pi^2}{N_c J_1 G_V}, \quad m_a^2 = m_\rho^2 + 6(m - \hat{m})^2. \quad (45)$$

In particular it implies the relations

$$g_A = 1 - \frac{6(m - \hat{m})^2}{m_a^2} = \frac{m_\rho^2}{m_a^2}, \quad \kappa = \frac{3}{m_a^2}. \quad (46)$$

We also have to redefine the spin-0 fields

$$\sigma = \sqrt{\frac{4\pi^2}{N_c J_1}} \sigma^{(ph)} \equiv g_\sigma \sigma^{(ph)}, \quad \pi = g_\pi \pi^{(ph)}, \quad g_\pi = \frac{g_\sigma}{\sqrt{g_A}}. \quad (47)$$

The mass formulae for spin-0 fields are

$$m_\pi^2 = \frac{\hat{m} g_\pi^2}{(m - \hat{m}) G_S}, \quad m_\sigma^2 = m_\pi^2 + 4(m - \hat{m})^2. \quad (48)$$

As compared with previous calculations in [4, 7, 15] our mass formulae have a different dependence on the current quark mass. Numerically these lead to small deviations in the final results for $\hat{m} \sim 7$ MeV. However in the case of broken $SU(3) \times SU(3)$ symmetry this effect is more essential and has to be taken into account with all care. Anyway, the small numerical difference between final results cannot justify the incorrect treatment of symmetry principles.

Let us also point out that after the field redefinitions the symmetry breaking pattern takes the form [25]

$$\delta \mathcal{L}_{\text{eff}} = -2m_\pi^2 f_\pi \beta_i \pi_i^{(ph)} \quad (49)$$

where we used the relation

$$g_\pi = \frac{m - \hat{m}}{f_\pi}. \quad (50)$$

Formula (49) leads to the well known PCAC relation for the divergence of the quark axial-vector current

$$\partial_\mu \vec{J}_5^\mu = 2f_\pi m_\pi^2 \vec{\pi}^{(ph)}. \quad (51)$$

5 The coset-space parametrization for meson fields

To compare our Lagrangian in full detail with the result of the nonlinear approach [15] one has to perform a chiral field dependent rotation which eliminates the nonderivative coupling of π . A systematic treatment of the problem has been developed by Coleman, Wess and Zumino [31]. We consider here the approximation to this picture which is known as the nonlinear realization in which no scalar particles exist, i.e. we shall eliminate completely the scalar degree of freedom from the meson Lagrangian (39). The conventional method to realize this idea is based on the fact that the sum $(\sigma + m - \hat{m})^2 + \vec{\pi}^2$ is invariant under chiral transformations. Therefore one can put it equal to a constant (the nonlinear ansatz) without spoiling chiral symmetry. In this case the scalar field σ is no more an independent variable and can be excluded from the Lagrangian [32, 25] in favour of the pion field. A constant can be fixed at a point $\langle \sigma \rangle = \langle \pi_i \rangle = 0$, i.e.

$$(\sigma + m - \hat{m})^2 + \vec{\pi}^2 = (m - \hat{m})^2. \quad (52)$$

The theory with the constraint (52) can be formulated in terms of the generating functional

$$\begin{aligned} Z_1 = & \int \mathcal{D}q \mathcal{D}\bar{q} \mathcal{D}\sigma \mathcal{D}\pi_i \mathcal{D}v_\mu^i \mathcal{D}a_\mu^i \mathcal{D}\lambda \exp \left\{ i \int d^4x \left[\mathcal{L}_{\text{qm}} \right. \right. \\ & \left. \left. - \frac{\lambda}{2} \left((\sigma + m - \hat{m})^2 + \vec{\pi}^2 - (m - \hat{m})^2 \right) \right] \right\} \end{aligned} \quad (53)$$

where \mathcal{L}_{qm} is the quark-meson Lagrangian (15) rewritten in the broken phase (the replacement $\sigma \rightarrow \sigma + m$ is done). One has to take an integral over the Lagrange multiplier $\lambda(x)$. It leads to the δ -functional

$$\delta \left[(\sigma + m - \hat{m})^2 + \vec{\pi}^2 - (m - \hat{m})^2 \right] = \sum_{a=1}^2 \frac{\delta(\sigma - \sigma_a)}{2\sqrt{(m - \hat{m})^2 - \vec{\pi}^2}} \quad (54)$$

where

$$\sigma_{1,2} = (m - \hat{m}) \left(\pm \sqrt{1 - \frac{\vec{\pi}^2}{(m - \hat{m})^2}} - 1 \right). \quad (55)$$

In Z_1 one has to integrate only the configurations σ with $\langle \sigma \rangle = 0$. It means that only σ_1 contributes to the generating functional, for $\langle \sigma_2 \rangle \neq 0$. After integrating over σ and quark fields we have

$$Z_1 = \int \mathcal{D}\mu[\pi_i] \mathcal{D}v_\mu^i \mathcal{D}a_\mu^i \exp \left\{ i \int d^4x \mathcal{L}'_{\text{eff}}(\pi_i, v_\mu^i, a_\mu^i) \right\}. \quad (56)$$

The Lagrangian $\mathcal{L}'_{\text{eff}}(\pi_i, v_\mu^i, a_\mu^i)$ is our Lagrangian (39) where one has to substitute σ by σ_1 . The $SU(2) \times SU(2)$ -invariant measure $\mathcal{D}\mu[\pi_i]$ emerging in Z_1 is related with the curvature in the space of π_i variables,

$$\prod_{i=1}^3 \mathcal{D}\mu[\pi_i] = \prod_{i=1}^3 \frac{\mathcal{D}\pi_i}{\sqrt{1 - \frac{\pi^2}{(m-\hat{m})^2}}}. \quad (57)$$

It is more convenient now to introduce new variables ϕ_i , different from π_i , defined as

$$\pi_i = (m - \hat{m}) \frac{\phi_i}{\phi} \sin \phi, \quad \phi = \sqrt{\phi_i^2}. \quad (58)$$

The parametrization in terms of ϕ_i fields corresponds to the normal coordinate system on the surface of a three-dimensional sphere: the $SU(2) \times SU(2)/SU(2)$ group manifold. One can get the expression for the invariant measure in these new variables

$$\prod_{i=1}^3 \mathcal{D}\mu[\pi_i] = \prod_{i=1}^3 \det \left(\frac{\partial \pi_n}{\partial \phi_m} \right) \frac{\mathcal{D}\phi_i}{\cos \phi} = N \prod_{i=1}^3 \frac{\sin^2 \phi}{\phi^2} \mathcal{D}\phi_i. \quad (59)$$

We drop here the expression for the nonessential factor N .

It is not difficult to get from (21) the form of the infinitesimal chiral transformation for the variables ϕ_i

$$\delta \phi_i = 2\beta_i \phi \cot \phi + 2 \frac{\phi_i \phi_k}{\phi^2} \beta_k (1 - \phi \cot \phi) - 2\epsilon_{ijk} \alpha_j \phi_k. \quad (60)$$

Geometrically this transformation is nothing else than a law for the coordinate changes in the $SU(2) \times SU(2)/SU(2)$ coset-space under the action of the $SU(2) \times SU(2)$ chiral group. Let us rewrite the Lagrangian $\mathcal{L}'_{\text{eff}}(\pi_i, v_\mu^i, a_\mu^i)$ in terms of these new variables. It is customary to put it in the form which

includes the fields with the covariant transformation law. For this purpose one has to introduce also the new vector and axial-vector variables

$$\begin{aligned} v_\mu &= \frac{1}{2} [\xi^\dagger(v'_\mu + a'_\mu)\xi + \xi(v'_\mu - a'_\mu)\xi^\dagger] \\ a_\mu &= \frac{1}{2} [\xi^\dagger(v'_\mu + a'_\mu)\xi - \xi(v'_\mu - a'_\mu)\xi^\dagger]. \end{aligned} \quad (61)$$

We use here the standard definition of the coset representative ξ

$$\xi = \exp\left(-\frac{i}{2}\tau_i\phi_i\right). \quad (62)$$

One can show that the Jacobian of this replacement is equal to one

$$\frac{\partial(v_{i\mu}, a_{i\mu})}{\partial(v'_{j\mu}, a'_{j\mu})} = 1 \quad (63)$$

and the transformation laws of new v'_μ, a'_μ fields are covariant, i.e.

$$\delta v'_\mu = i[\alpha + \beta(\phi), v'_\mu], \quad \delta a'_\mu = i[\alpha + \beta(\phi), a'_\mu] \quad (64)$$

where

$$\beta(\phi) = \beta_k(\phi)\tau_k, \quad \beta_k(\phi) = \epsilon_{kni}\beta_i\frac{\phi_n}{\phi} \tan \frac{\phi}{2}. \quad (65)$$

Let us note that the part of chiral transformations which depends on the parameter β is x -dependent now, $\beta(\phi)$, through the field $\phi(x)$. We remind also with the purpose of future references that the function ξ_μ has the same transformation law

$$\xi_\mu = -i(\xi\partial_\mu\xi^\dagger - \xi^\dagger\partial_\mu\xi), \quad \delta\xi_\mu = i[\alpha + \beta(\phi), \xi_\mu]. \quad (66)$$

Another function, Γ_μ , defines the covariant derivative in the coset space, i.e. if R transforms covariantly then the same is true for its covariant derivative $d_\mu R$ defined by

$$d_\mu R = \partial_\mu R + [\Gamma_\mu, R] \quad (67)$$

where

$$\Gamma_\mu = \frac{1}{2} (\xi\partial_\mu\xi^\dagger + \xi^\dagger\partial_\mu\xi). \quad (68)$$

One can show that Γ_μ transforms like Yang – Mills connection on the given coset space

$$\delta\Gamma_\mu = i[\alpha + \beta(\phi), \Gamma_\mu] - i\partial_\mu\beta(\phi). \quad (69)$$

All these functions appear naturally in the effective Lagrangian (56) by means of the abovementioned replacements of variables and we have as a result

$$\begin{aligned} \mathcal{L}'_{\text{eff}} &= \frac{1}{4G_V} \text{tr} [(v'_\mu)^2 + (a'_\mu)^2] + \frac{m_\pi^2}{4} f_\pi^2 \text{tr} (\xi\xi + \xi^\dagger\xi^\dagger - 2) \\ &+ \frac{N_c J_1}{16\pi^2} \left[(m - \hat{m})^2 \text{tr} (\xi_\mu - 2a'_\mu)^2 - \frac{1}{3} \text{tr} (\tilde{V}_{\mu\nu}^2 + \tilde{A}_{\mu\nu}^2) \right] \end{aligned} \quad (70)$$

where we put

$$\tilde{V}_{\mu\nu} = \mathcal{V}'_{\mu\nu} + \frac{i}{2} ([\xi_\mu, a'_\nu] - [\xi_\nu, a'_\mu]), \quad (71)$$

$$\tilde{A}_{\mu\nu} = \mathcal{A}'_{\mu\nu} + \frac{i}{2} ([\xi_\mu, v'_\nu] - [\xi_\nu, v'_\mu]), \quad (72)$$

and

$$\mathcal{V}'_{\mu\nu} = d_\mu v'_\nu - d_\nu v'_\mu - i[v'_\mu, v'_\nu] - i[a'_\mu, a'_\nu], \quad (73)$$

$$\mathcal{A}'_{\mu\nu} = d_\mu a'_\nu - d_\nu a'_\mu - i[a'_\mu, v'_\nu] - i[v'_\mu, a'_\nu]. \quad (74)$$

Similary to the case with linear realization of chiral symmetry one has now to diagonalize the pseudoscalar – axial-vector bilinear form in the Lagrangian (70). The replacement

$$a'_\mu \rightarrow a'_\mu + \kappa(m - \hat{m})^2 \xi_\mu \quad (75)$$

with κ defined by (42) solves the problem. We have as a result

$$\begin{aligned} \mathcal{L}'_{\text{eff}} &= \frac{f_\pi^2}{4} \text{tr} (\xi_\mu \xi_\mu) + \frac{m_\pi^2}{4} f_\pi^2 \text{tr} (\xi\xi + \xi^\dagger\xi^\dagger - 2) \\ &- \frac{1}{2g_\rho^2} \text{tr} (V_{\mu\nu}^2 + A_{\mu\nu}^2) + \frac{1}{g_\rho^2} \text{tr} [m_\rho^2 (v'_\mu)^2 + m_a^2 (a'_\mu)^2] \end{aligned} \quad (76)$$

where after the replacement (75) the antisymmetric tensors $V_{\mu\nu}$ and $A_{\mu\nu}$ read

$$V_{\mu\nu} = \mathcal{V}'_{\mu\nu} + \frac{ig_A}{2} ([\xi_\mu, a'_\nu] - [\xi_\nu, a'_\mu]) + \frac{i}{4} (1 - g_A^2) [\xi_\mu, \xi_\nu], \quad (77)$$

$$A_{\mu\nu} = \mathcal{A}'_{\mu\nu} + \frac{ig_A}{2} ([\xi_\mu, v'_\nu] - [\xi_\nu, v'_\mu]) + \frac{1}{2} (1 - g_A) (d_\mu \xi_\nu - d_\nu \xi_\mu). \quad (78)$$

The following redefinitions lead us to the physical pseudoscalar, vector, and axial-vector states

$$\phi_i = \frac{1}{f_\pi} \pi_i^{(ph)}, \quad v'_\mu = \frac{g_\rho}{2} v_\mu^{(ph)}, \quad a'_\mu = \frac{g_\rho}{2} a_\mu^{(ph)}. \quad (79)$$

The Lagrangian (76) is the result of the nonlinear realization in which no scalar particles exist. It is sufficient to illustrate our point, although it is an approximation. The first term in (76) is the canonical Lagrangian for the nonlinear sigma model. The second term of the Lagrangian breaks chiral symmetry and obviously satisfies the symmetry breaking pattern of the basic quark Lagrangian. Except for that term the rest of the terms in (76) are manifestly chiral invariant. It means that all \hat{m} -dependence is absorbed in coupling constants, for instance the mass of the axial-vector meson, m_a , and the coupling, g_A , depend on \hat{m} (see eq.(45) and eq.(46)). Our expression (76) clearly shows that only the symmetry breaking part of the Lagrangian includes the cluster $\Sigma = \xi^\dagger \xi^\dagger + \xi \xi$ with a noncovariant transformation law. This combination (and the other similar one: $\Delta = \xi^\dagger \xi^\dagger - \xi \xi$) never appears among the interaction vertices, it would generate spurious symmetry breaking effects. The structure of our Lagrangian differs in this respect from the known expressions where the explicit symmetry breaking effect has also been included (see for instance [33, 15]).

6 Concluding remarks

The modulus of the chiral fermion determinant is well defined by the formula $\ln |\det D|$. It has generally been assumed that this formula can be also used in the case when chiral symmetry is explicitly broken. In this work we have shown in considerable detail that it is not true. We have described a practical tool to derive in a systematic and consistent way the real part of $\ln \det D$ considering as an example the ENJL model with the chiral $SU(2) \times SU(2)$ symmetric four-quark interactions. In this special case we arrive at the fermion determinant as a result of integration over quark fields in the corresponding generating functional. The effective meson Lagrangian describing the dynamics of collective degrees of freedom emerges in this way. The symmetry breaking pattern of the starting quark Lagrangian should not be changed during bosonization. This symmetry requirement together with

the Schwinger – Dyson equation which defines the vacuum state of the model helps us to fix completely the \hat{m} -dependent part of the effective Lagrangian. It differs from the ones obtained on the ground of other methods like the direct calculation of the one-loop Feynman diagrams, or the naive use of the proper-time representation in form (1).

In this work we have addressed another point also related with the chiral symmetry transformation laws. It is associated with the well-known mixing terms in the pseudoscalar-axialvector fields. We have shown that the way presented in the literature to diagonalize this admixture in the case of the linear realization of chiral symmetry is not compatible with the transformation laws for the vector and axial-vector fields and induces spurious symmetry breaking terms. We have derived the minimal form of covariant redefinitions for the spin-1 fields needed to fulfill simultaneously the mesonic transformation laws and diagonalization. We find out that our redefinitions of vector and axial-vector fields is an agreement with the standard redefinition of axial-vector fields in the nonlinear case.

In the end we have rewritten our Lagrangian in the nonlinear form excluding completely the scalar field. This approximation is sufficient to pin down the general structures containing the explicit symmetry breaking terms in the effective mesonic Lagrangian for the ENJL model with nonlinear realization of chiral symmetry. We conclude that the effect of explicit chiral symmetry breaking has never been treated with enough care in the framework of the NJL model and have presented in this work a consistent method to take it into account.

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Appendix

Let us show here that the fields redefinition (41) coincides with a similar redefinition which has been used in the case of non-linear realization of chiral symmetry. We shall start from the piece of Lagrangian (15) with the

collective fields of spin-1

$$\bar{q}\gamma^\mu(v_\mu + \gamma_5 a_\mu)q = \bar{q}\gamma^\mu \left\{ v'_\mu + i\frac{\kappa}{2}[\pi, \partial_\mu \pi] + \gamma_5 \left[a'_\mu + \kappa((\sigma + m - \hat{m})\partial_\mu \pi - \pi\partial_\mu \sigma) \right] \right\} q. \quad (80)$$

To come to the non-linear realization of chiral symmetry one has to eliminate the scalar field, which is achieved by the constraint

$$\sigma + m - \hat{m} = (m - \hat{m}) \sqrt{1 - \frac{\pi_i^2}{(m - \hat{m})^2}}. \quad (81)$$

Let us choose the exponential parametrization for pion fields

$$\pi_i = (m - \hat{m}) \frac{\phi_i}{\phi} \sin \phi, \quad \phi = \sqrt{\phi_i^2}. \quad (82)$$

The pure geometrical picture appears if we redefine at the same time the quark fields

$$Q = (\xi^\dagger P_R + \xi P_L)q, \quad \xi = \exp\left(-\frac{i}{2}\tau_i \phi_i\right) \quad (83)$$

where the projection operators $2P_R = (1 + \gamma_5)$ and $2P_L = (1 - \gamma_5)$ have been introduced. In this case one needs also to redefine the vector and axial-vector fields. The new variables V_μ and A_μ are the following ones

$$\begin{aligned} V_\mu &= \frac{1}{2} \left[\xi(v'_\mu + a'_\mu)\xi^\dagger + \xi^\dagger(v'_\mu - a'_\mu)\xi \right], \\ A_\mu &= \frac{1}{2} \left[\xi(v'_\mu + a'_\mu)\xi^\dagger - \xi^\dagger(v'_\mu - a'_\mu)\xi \right]. \end{aligned} \quad (84)$$

In these variables the replacement can be written as

$$\begin{aligned} V_\mu &\rightarrow V_\mu + (m - \hat{m})^2 \frac{\kappa}{2} \left[\xi(X_\mu + Y_\mu)\xi^\dagger + \xi^\dagger(X_\mu - Y_\mu)\xi \right], \\ A_\mu &\rightarrow A_\mu + (m - \hat{m})^2 \frac{\kappa}{2} \left[\xi(X_\mu + Y_\mu)\xi^\dagger - \xi^\dagger(X_\mu - Y_\mu)\xi \right] \end{aligned} \quad (85)$$

where $X_\mu = \tau_k X_{k\mu}$, $Y_\mu = \tau_k Y_{k\mu}$ with the following expressions for $X_{k\mu}$ and $Y_{k\mu}$

$$X_{k\mu} = \varepsilon_{kji} \phi_i \partial_\mu \phi_j \frac{\sin^2 \phi}{\phi^2}, \quad (86)$$

$$Y_{k\mu} = \left[\delta_{kj} \sin \phi \cos \phi + \frac{\phi_k \phi_j}{\phi} \left(1 - \frac{\sin \phi}{\phi} \cos \phi \right) \right] \frac{\partial_\mu \phi_j}{\phi}. \quad (87)$$

One can obtain that

$$\xi(X_\mu + Y_\mu)\xi^\dagger + \xi^\dagger(X_\mu - Y_\mu)\xi = 0, \quad (88)$$

$$\xi(X_\mu + Y_\mu)\xi^\dagger - \xi^\dagger(X_\mu - Y_\mu)\xi = 2\xi_\mu \quad (89)$$

where

$$\xi_\mu = \tau_k \xi_{k\mu}, \quad \xi_{k\mu} = \left[\delta_{kj} \frac{\sin \phi}{\phi} + \frac{\phi_k \phi_j}{\phi^2} \left(1 - \frac{\sin \phi}{\phi} \right) \right] \partial_\mu \phi_j. \quad (90)$$

Therefore we get for the nonlinear case the standard replacement for the axial-vector field

$$A_\mu \rightarrow A_\mu + \kappa(m - \hat{m})^2 \xi_\mu. \quad (91)$$

At the same time the vector field does not change.

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